MATH ANALYSIS--TRIGONOMETRY

LEARNING ACTIVITIES PACKAGE

TRIGONOMETRIC FUNCTIONS

BEHAVIORAL OBJECTIVES

I. Define

A. sine x; sin x
B. cosine x; cos x
C. tangent x; tan x
D. cosecant x; csc x
E. secant x; sec x
F. cotangent x; cot x

II. For each of the trigonometric functions determine

A. The domain of the function
B. The range of the function
C. The zeros of the function
D. The monotonicity of the function over a specified domain
E. The period of the function
F. The equations of any assymptotes of the graph of the function
G. Whether the function is odd or even

III. Sketch the graph of each of the six trigonometric functions

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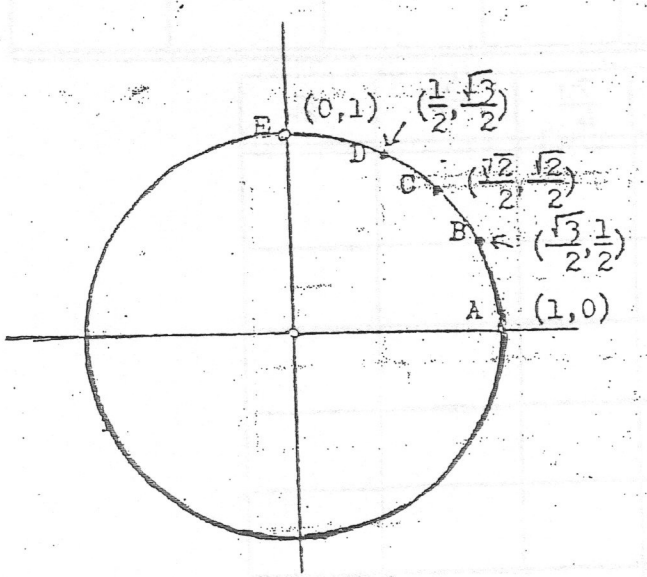
SECTION I THE TRIGONOMETRIC FUNCTIONS--DEFINED

Consider the unit circle. From the unit circle the values of each of the six trigonometric functions are defined.

UNIT CIRCLE



DEFINITIONS:

$W(\theta) = (x,y)$

All six trig functions are defined in terms of the ordered pairs on the unit circle.

$\sin\theta = y$	$\csc\theta = \frac{1}{y}$
$\cos\theta = x$	$\sec\theta = \frac{1}{x}$
$\tan\theta = \frac{y}{x}$	$\cot\theta = \frac{x}{y}$

EXAMPLES:

$\sin 0 = 0$ $\quad \cos 0 = 1$ $\quad \sin \frac{\pi}{6} = \frac{1}{2}$ $\quad \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\sec \frac{\pi}{3} = 2$ $\quad \cos \pi = -1$ $\quad \tan \frac{3\pi}{4} = -1$ $\quad \sin -\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

The first exercise of this L.A.P. concerns itself with evaluating the trig functions for given values of θ. The management suggests that the student first draw a unit circle and label the desired points. . . see page 1. Complete the values (x,y) for each of the quadrants.

Note: for a function like sec $\frac{\pi}{6}$, the value is $\frac{1}{\frac{\sqrt{3}}{2}}$. It is not good form to leave the expression in this form. First it becomes $\frac{2}{\sqrt{3}}$, then, by rationalizing, the expression is: $\frac{2\sqrt{3}}{3}$

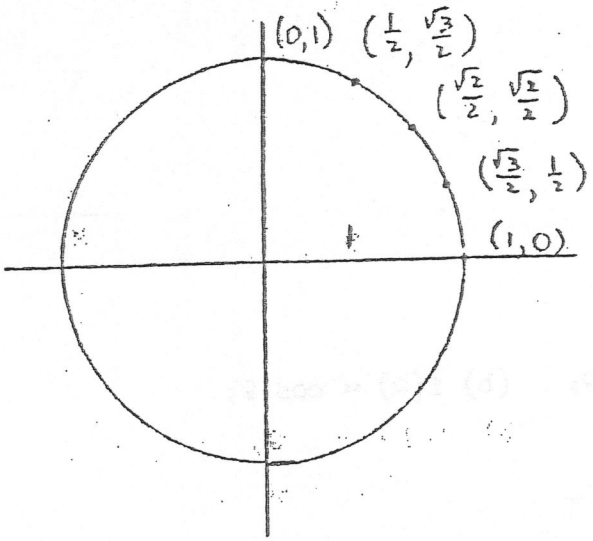
EXERCISE 1 Complete the following tables:

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
sin									
cos									
tan									
cot									
sec									
csc									

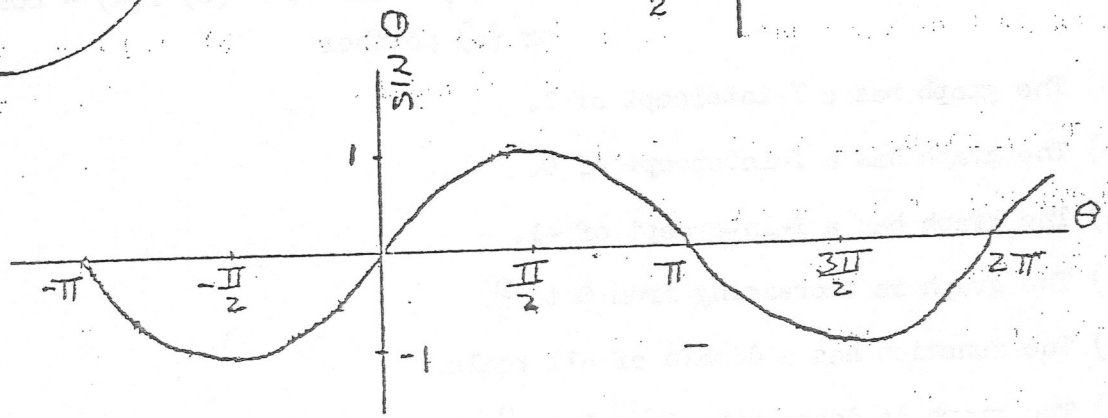
θ	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
sin								
cos								
tan								
cot								
sec								
csc								

SECTION 2

SIN AND COS GRAPHS



θ	sun θ
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$
$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1



As θ increases from 0 to $\frac{\pi}{2}$, sin θ increases from 0 to 1.

As θ increases from $\frac{\pi}{2}$ to π, to 2π, sin θ moves from 1 to 0 to -1 and back to 0.

And so it goes, on and on, ever moving regularly between 1 and -1. The curve is smooth, reaching a maximum of 1 and a minimum of -1. The sin function is <u>periodic</u> with a period of 2π. Every 2π units, the values of the function are repeated. $\sin(\theta + 2k\pi) = \sin \theta$.

The amplitude of a periodic function is $\frac{1}{2}$(maximum value - minimum value).

The amplitude of the function $f(\theta) = \sin \theta$ is 1.

The domain of the sin function is $(-\infty, \infty)$. The range of the sin function is $[-1,1]$. The sin function is an odd function. Notice: $\sin(-\theta) = -\sin \theta$.

Notice also, the graph of the sin function demonstrates origin symmetry.

EXERCISE 2

1. Graph the function $f(\theta) = \cos\theta$, for $-\pi \leq \theta \leq 3\pi$.

2. What is the domain of $f(\theta) = \cos\theta$?

3. What is the range of $f(\theta) = \cos\theta$?

4. What is the amplitude of $f(\theta) = \cos\theta$?

5. What is the period of $f(\theta) = \cos\theta$?

6. Is the function $f(\theta) = \cos\theta$ odd or even?

7. Graph the function $f(\theta) = \sin\theta$ for $-\pi \leq \theta \leq 3\pi$.

8. Answer each of the following with (a) $f(\theta) = \sin\theta$; (b) $f(\theta) = \cos\theta$; or (c) neither

(a) The graph has a Y-intercept of 1.

(b) The graph has a Y-intercept of 0.

(c) The graph has a Y-intercept of -1.

(d) The graph is increasing from 0 to $\frac{\pi}{2}$.

(e) The function has a domain of all reals.

(f) The graph is decreasing from 0 to $\frac{\pi}{2}$.

(g) The function has a maximum value at $\theta = \frac{\pi}{2}$.

(h) The function has a maximum value at $\theta = 0$.

(i). The function has a period of 2π.

(j) The graph of the function has X-intercepts at $\theta = k\pi$.

(k) The graph of the function has X-intercepts at $\theta = \frac{(2k+1)\pi}{2}$.

(l) The graph of the function exhibits Y- axis symmetry.

(m) The graph of the function exhibits X-axis symmetry.

(n) The graph of the function exhibits origin symmetry.

(o) The function is an odd function.

(p) The function is an even function.

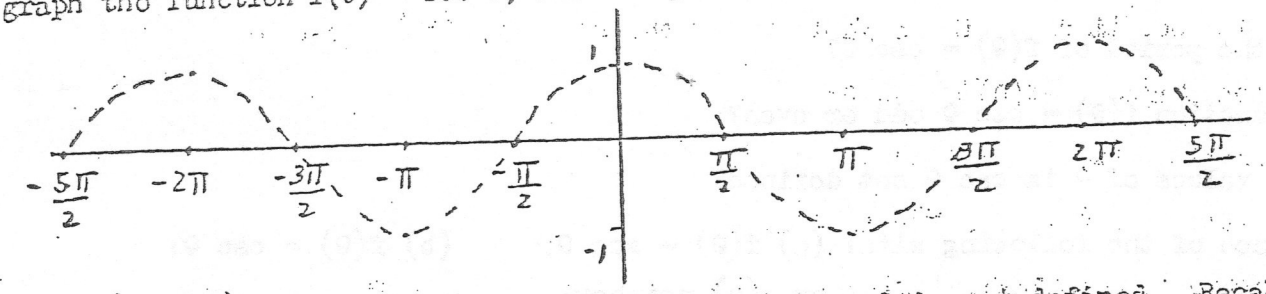
(q) $f(\frac{\pi}{2}) = 0$; (r) $f(\pi) = -1$ (s) $f(2\pi) = 1$

(t) $f(\frac{\pi}{3}) = \frac{1}{2}$ (u) $f(-\pi) = 0$ (v) $f(-\frac{\pi}{2}) = -1$

ECTION 3 SECANT AND COSECANT GRAPHS

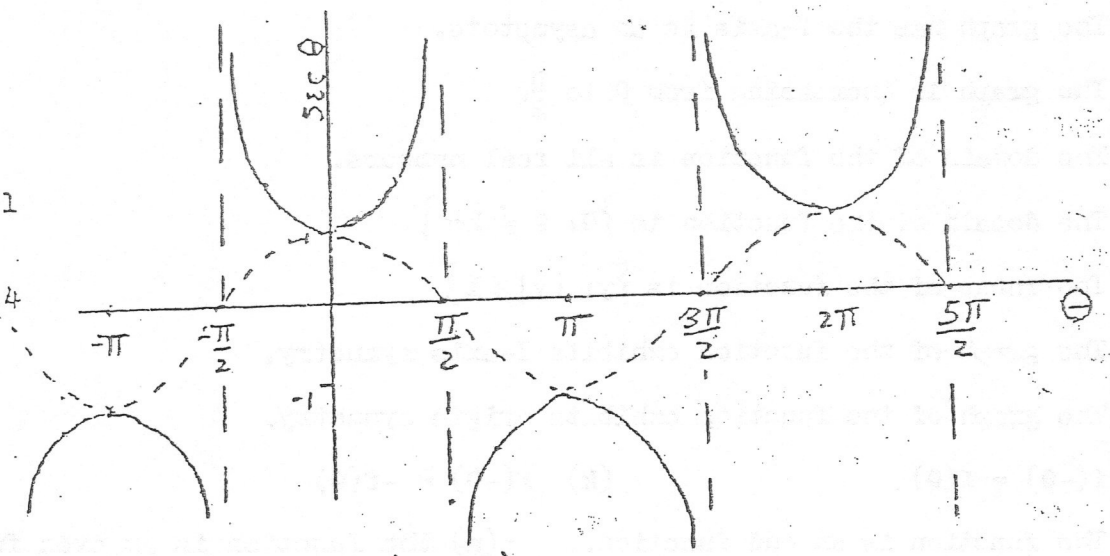
he values of the secant function are the reciprocals of thevalues of the cosine unction. For every θ such that $\cos\theta \neq 0$, $\sec\theta = \frac{1}{\cos\theta}$.

o graph the function $f(\theta) = \sec\theta$, first lightly sketch $f(\theta) = \cos\theta$



or $\theta = \frac{(2k+1)\pi}{2}$, $\cos\theta = 0$. For these values, $\sec\theta$ is not defined. Recall, is not defined. Consider the values of $f(\theta) = \cos\theta$ and plot the reciprocals for nough values to sketch the curve for $0 < \theta < \frac{\pi}{2}$. For these values, $\sec\theta$ moves rom 1 to $+\infty$.

θ	$\sec\theta$
0	1
$\frac{\pi}{6}$	$\frac{2\sqrt{3}}{3} \doteq 1.1$
$\frac{\pi}{4}$	$\sqrt{2} \doteq 1.4$
$\frac{\pi}{3}$	2
$\frac{\pi}{2}$	---



The domain of $f(\theta) = \sec\theta$ is $\left\{\theta : \theta \neq \frac{(2k+1)\pi}{2}\right\}$.

The range of $f(\theta) = \sec\theta$ is $\{y : |y| \geq 1\}$.

$f(\theta) = \sec\theta$ is an even function. Notice, $f(-\theta) = f(\theta)$. Also notice that the graph of the function $f(\theta) = \sec\theta$ exhibits Y-axis symmetry.

As the function $f(\theta) = \cos\theta$ is periodic with a period of 2π, so, $f(\theta) = \sec\theta$ is also periodic with a period of 2π.

Since $f(\theta) = \sec\theta$ has no maximum value and no minimum value, the notion of amplitude is not applicable with respect to this function.

The lines with the equations $\theta = \frac{(2k+1)\pi}{2}$ are asymptotes for the graph of the function $f(\theta) = \sec\theta$.

EXERCISE 3

1. Graph the function $f(\theta) = \csc \theta$, for $-\pi < \theta < 3\pi$.

2. What is the domain of $f(\theta) = \csc \theta$?

3. What is the range of $f(\theta) = \csc \theta$?

4. What are the equations of the asymptotes of the graph of $f(\theta) = \csc \theta$?

5. What is the period of $f(\theta) = \csc \theta$?

6. Is the function $f(\theta) = \csc \theta$ odd or even?

7. For what values of θ is $\csc \theta$ not defined?

8. Answer each of the following with: (a) $f(\theta) = \sec \theta$; (b) $f(\theta) = \csc \theta$; or (c) neither.

(a) The graph has a Y-intercept of 1.

(b) The graph has a Y-intercept of 0.

(c) The graph has the Y-axis as an asymptote.

(d) The graph is increasing from 0 to $\frac{\pi}{2}$.

(e) The domain of the function is all real numbers.

(f) The domain of the function is $\{\theta: \theta \neq k\pi\}$.

(g) The range of the function is $\{y: |y| \geq 1\}$

(h) The graph of the function exhibits Y-axis symmetry.

(i) The graph of the function exhibits origin symmetry.

(j) $f(-\theta) = f(\theta)$ (k) $f(-\theta) = -f(\theta)$

(l) The function is an odd function. (m) The function is an even function.

(n) The function has a period of 2π.

(o) The graph of the function has no X-intercepts.

(p) The domain of the function is $\left\{\theta: \theta \neq \frac{(2k+1)\pi}{2}\right\}$

(q) $f(0) = 1$ (r) $f(\frac{\pi}{2}) = 1$ (s) $f(\frac{\pi}{4}) = \sqrt{2}$

(t) $f(\frac{\pi}{6}) = 2$ (u) $f(0)$ is undefined (v) $f(\pi) = 0$

(w) $f(\frac{\pi}{3}) = 2$ (x) $f(-\frac{\pi}{4}) = -\sqrt{2}$ (y) $f(\frac{\pi}{6}) = \frac{2\sqrt{3}}{3}$

SECTION 4

TANGENT AND COTANGENT GRAPHS

For ordered pairs (x,y) on the unit circle, $\tan\theta = \frac{y}{x}$, or $\tan\theta = \frac{\sin\theta}{\cos\theta}$.

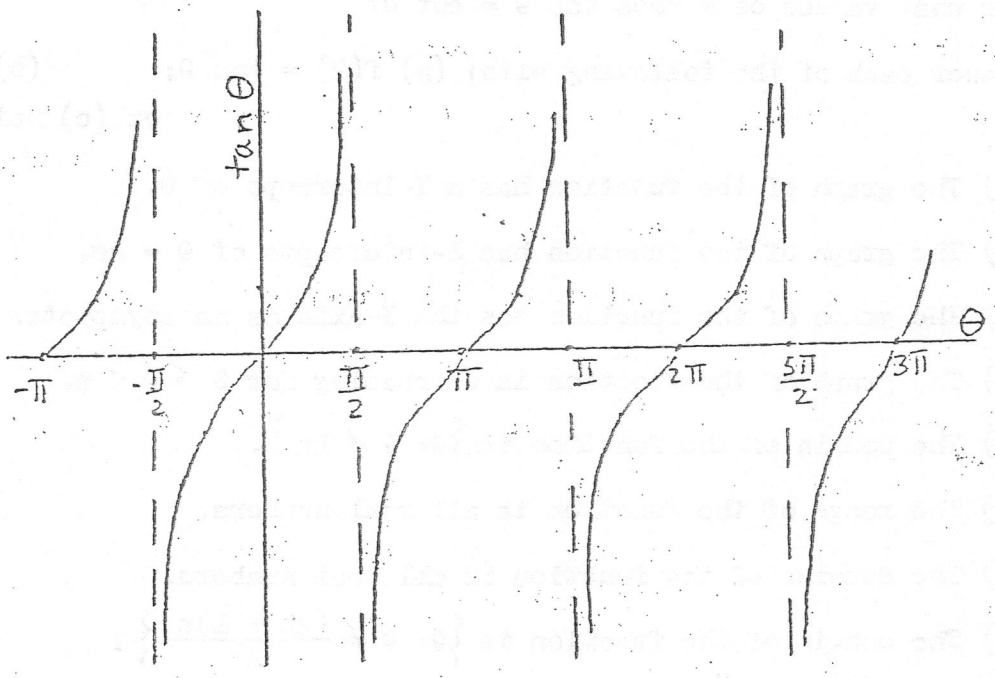
$f(\theta) = \tan\theta$ is defined for all θ such that $\theta \neq \frac{(2k+1)\pi}{2}$.

Notice, for $\theta = \frac{(2k+1)\pi}{2}$, the value of the x coordinate is 0.

As θ increases from 0 to $\frac{\pi}{2}$, x decreases and y increases. Hence, the ratio, $\frac{y}{x}$ increases.

With the unit circle at hand the following can be determined:

θ	$\tan\theta$
$-\frac{\pi}{2}$	---
$-\frac{\pi}{3}$	$-\sqrt{3} \doteq -1.7$
$-\frac{\pi}{4}$	-1
$-\frac{\pi}{6}$	$-\frac{\sqrt{3}}{3} \doteq -.58$
0	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{3} \doteq .58$
$\frac{\pi}{4}$	1
$\frac{\pi}{3}$	$\sqrt{3} \doteq 1.7$



For $\frac{(2k+1)\pi}{2} < \theta < \frac{(2k+3)\pi}{2}$, $\tan\theta$ is an increasing function.

The graph of $f(\theta) = \tan\theta$ has asymptotes for $\theta = \frac{(2k+1)\pi}{2}$.

The domain of the tan function is $\left\{\theta: \theta \neq \frac{(2k+1)\pi}{2}\right\}$.

The range of the tan function is $(-\infty, \infty)$.

$f(\theta) = \tan\theta$ is periodic with a period of π. Note: $\tan(\theta + k\pi) = \tan\theta$.

$f(\theta) = \tan\theta$ is an odd function. Notice, the graph demonstrates origin symmetry. Also notice: $\tan -\theta = -\tan\theta$.

The graph of the tan function has x-intercepts for $\theta = k\pi$.

Since $f(\theta) = \tan\theta$ has no maximum value and no minimum value, the notion of amplitude is not applicable with respect to this function.

EXERCISE 4

1. Graph the function $f(\theta) = \cot \theta$, for $-\pi < \theta < 3\pi$.

2. What is the domain of $f(\theta) = \cot \theta$?

3. What is the range of $f(\theta) = \cot \theta$?

4. What is the range of $f(\theta) = \tan \theta$?

5. What are the equations of the asymptotes of the function $f(\theta) = \cot \theta$?

6. Is the function $f(\theta) = \cot \theta$ odd or even?

7. For what values of θ is $f(\theta) = \cot \theta$ not defined?

8. For what values of θ does $\tan \theta = \cot \theta$?

9. Answer each of the following with: (a) $f(\theta) = \tan \theta$; (b) $f(\theta) = \cot \theta$; or (c) neither.

(a) The graph of the function has a Y-intercept of 0.

(b) The graph of the function has X-intercepts of $\theta = k\pi$.

(c) The graph of the function has the Y-axis as an anymptote.

(d) The graph of the function is decreasing for $0 < \theta < \pi$.

(e) The domain of the function is $\{\theta;\ \theta \neq k\pi\}$.

(f) The range of the function is all real numbers.

(g) The domain of the function is all real numbers.

(h) The domain of the function is $\left\{\theta;\ \theta \neq \frac{(2k+1)\pi}{2}\right\}$.

(i) The graph of the function exhibits Y-axis symmetry.

(j) The graph of the function exhibits origin symmetry.

(k) $f(-\theta) = f(\theta)$. (l) $f(-\theta) = -f(\theta)$.

(m) The function has X-intercepts of $\theta = \frac{(2k+1)\pi}{2}$.

(n) The function is increasing for $0 < \theta < \frac{\pi}{2}$.

(p) The graph of the function exhibits X-axis symmetry.

(q) $f(0) = 0$ (r) $f(\frac{\pi}{2}) = 1$ (s) $f(\frac{\pi}{6}) = 2$

(t) $f(\theta) = f(-\theta)$ (u) $f(0)$ is undefined (v) $f(\frac{\pi}{4}) = 1$

(w) $f(-\frac{\pi}{4}) = -1$ (x) $f(-\frac{\pi}{6}) = -\sqrt{3}$ (y) $f(k\pi) = 0$

ANSWERS

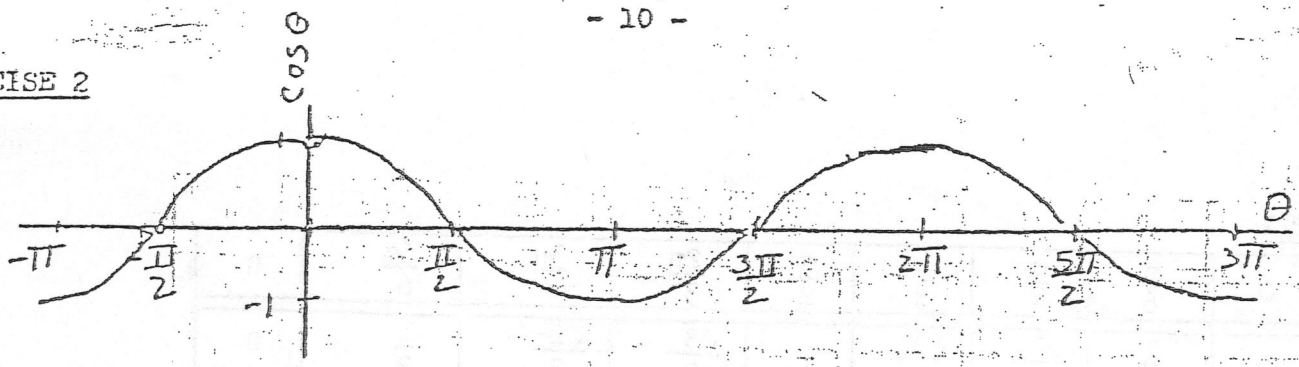
EXERCISE 1

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	---	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0
cot	---	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	---
sec	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	---	-2	$-\sqrt{2}$	$-\frac{2\sqrt{3}}{3}$	-1
csc	---	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	---

θ	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
sin	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0
cos	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
tan	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	---	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0
cot	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	---
sec	$-\frac{2\sqrt{3}}{3}$	$-\sqrt{2}$	-2	---	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1
csc	-2	$-\sqrt{2}$	$-\frac{2\sqrt{3}}{3}$	-1	$-\frac{2\sqrt{3}}{3}$	$-\sqrt{2}$	-2	---

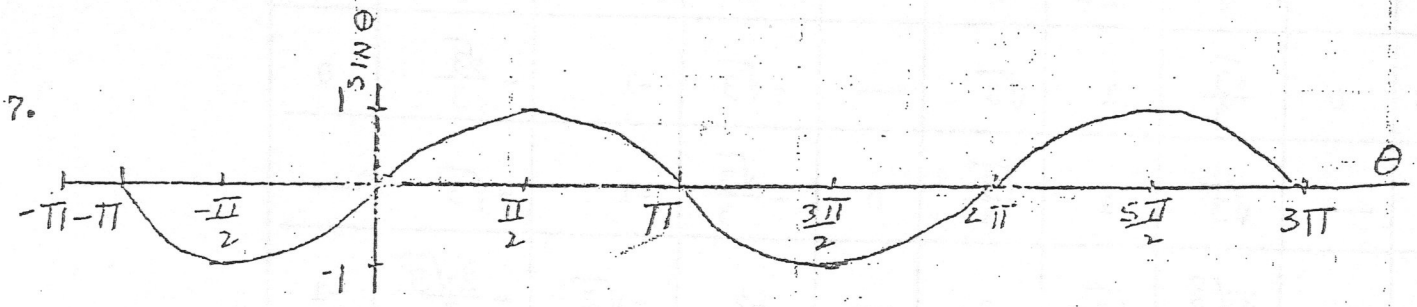
EXERCISE 2

1.



2. $(-\infty, \infty)$ 3. $[-1,1]$ 4. 1 5. 2π 6. Even

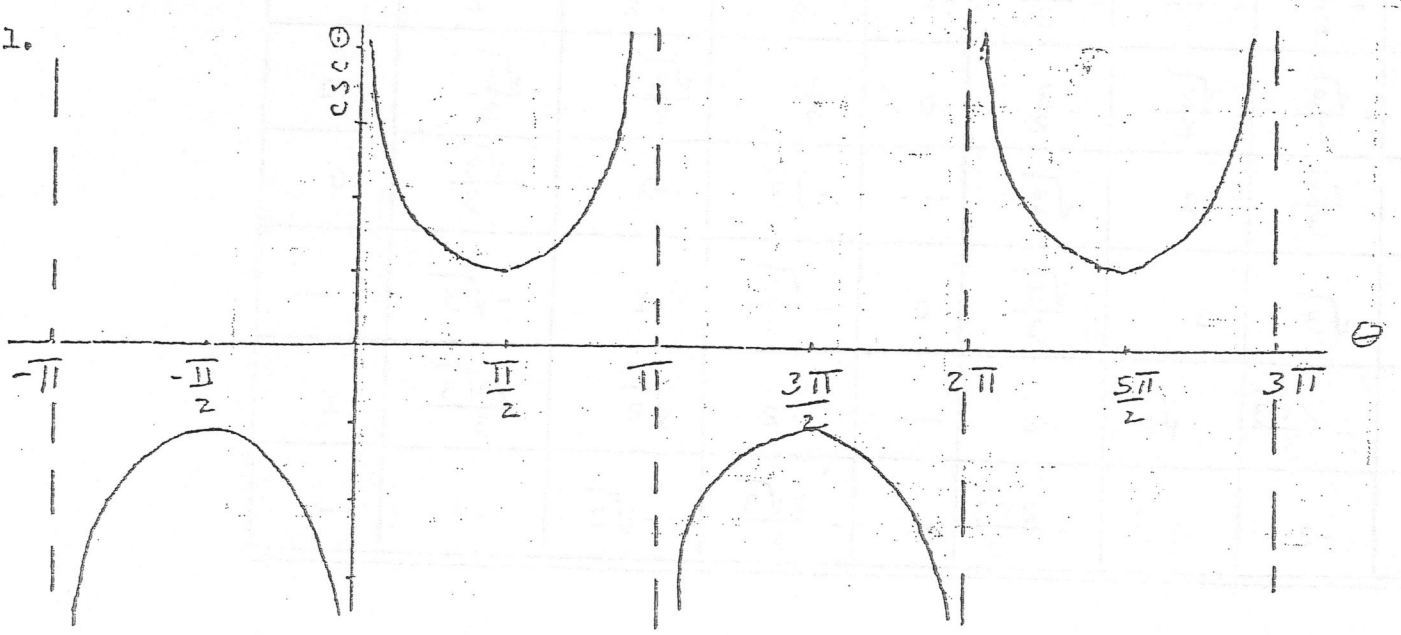
7.



8. (a) b; (b) a; (c) c; (d) a; (e) a and b; (f) b;
(g) a; (h) b; (i) a and b; (j) a; (k) b; (l) b; (m) c;
(n) a; (o) a; (p) b; (q) b; (r) b; (s) b; (t) b;
(u) a; (v) a

EXERCISE 3

1.



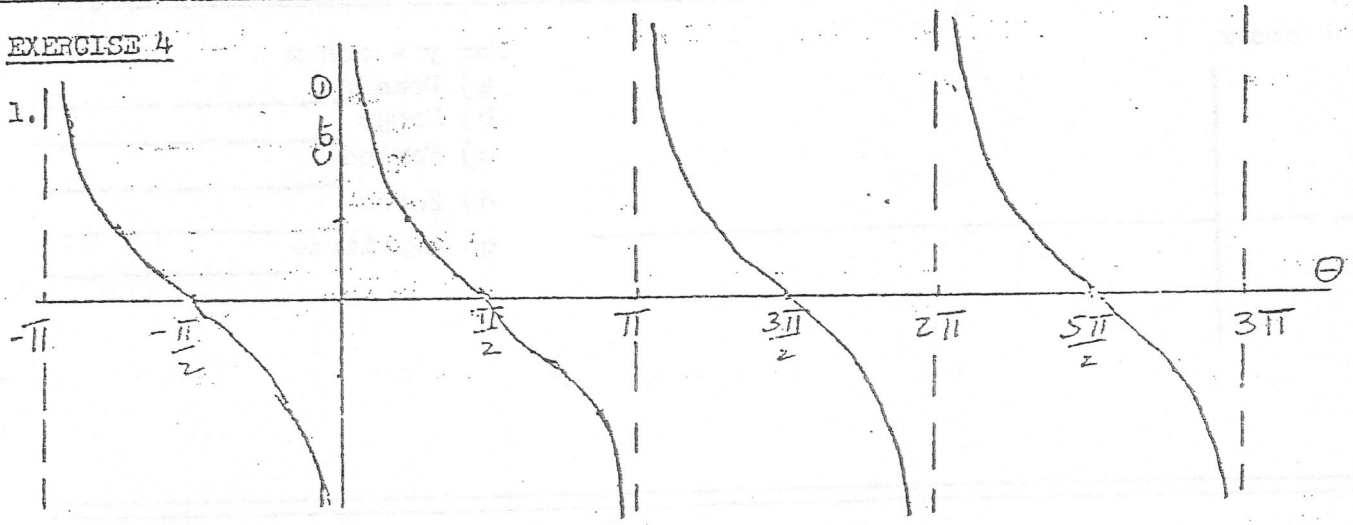
2. $\{\theta;\ \theta \neq k\pi\}$ 3. $\{y;\ |y| \geq 1\}$ 4. $\theta = k\pi$ 5. 2π

6. Odd 7. $\theta = k\pi$

8. (a) a; (b) c; (c) b; (d) a; (e) c (f) b;
(g) a and b; (h) a; (i) b; (j) a; (k) b; (l) b;
(m) a; (n) a and b; (o) a and b; (p) a; (q) a; (r) b;
(s) a and b; (t) b; (u) b; (v) c; (w) a; (x) b;
(y) a.

EXERCISE 4

1.



2. $\{\theta;\ \theta \neq k\pi\}$ 3. $(-\infty, \infty)$ 4. $(-\infty, \infty)$ 5. $\theta = k\pi$

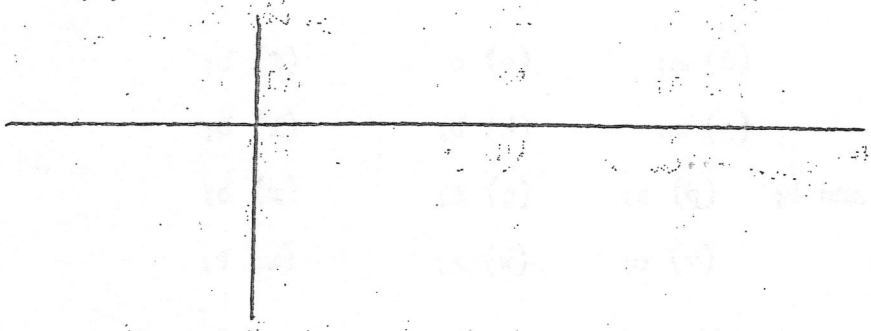
6. Odd 7. $\theta = k\pi$ 8. $\theta = \frac{(2k+1)\pi}{4}$

9. (a) a; (b) a; (c) b; (d) b; (e) b;
(f) a and b; (g) c; (h) a; (i) c; (j) a and b;
(k) c; (l) a and b; (m) b; (n) a; (p) c;
(q) a; (r) c; (s) c; (t) c; (u) b;
(v) a and b; (w) a and b; (x) b; (y) a

TRIGONOMETRIC FUNCTIONS TRIAL RUN

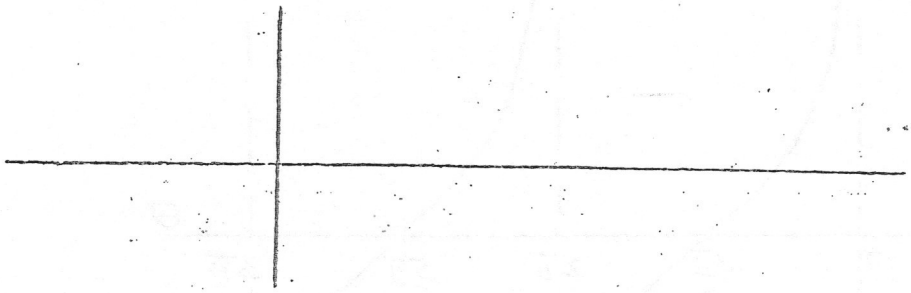
I Graph each of the following trig functions: Label the graphs carefully.
Graph at least two periods of each function.

1) $y = \sin x$



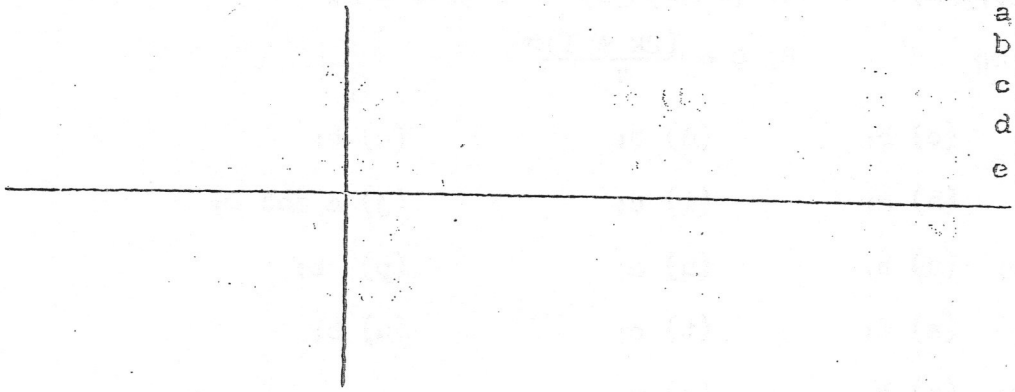
For $y = \sin x$:
a) Domain
b) Range
c) Period
d) Zeros
e) Amplitude

2) $y = \cos x$



For $y = \cos x$
a) Domain
b) Range
c) Period
d) Zeros
e) Amplitude

3) $y = \tan x$

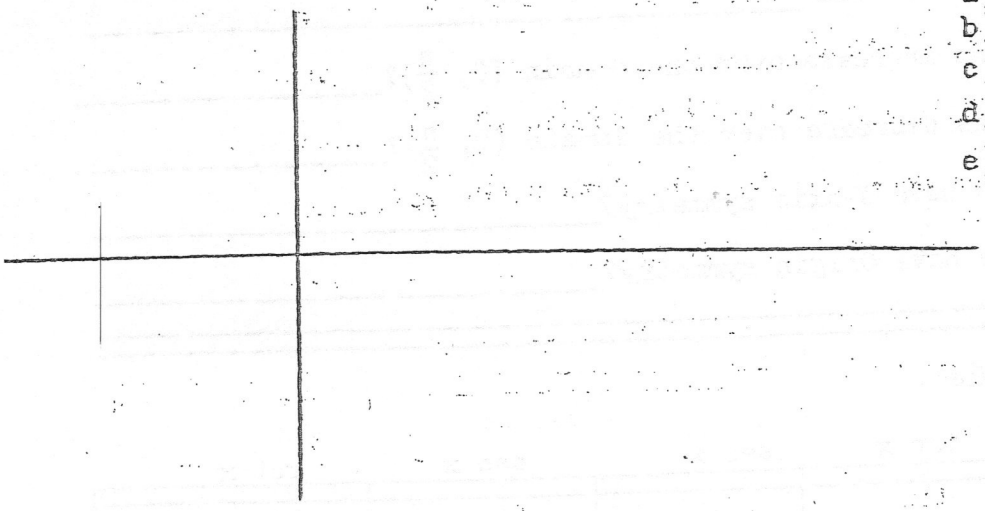


For $y = \tan x$
a) Domain
b) Range
c) Period
d) Zeros
e) Equation of assymptotes

TRIGONOMETRIC FUNCTIONS

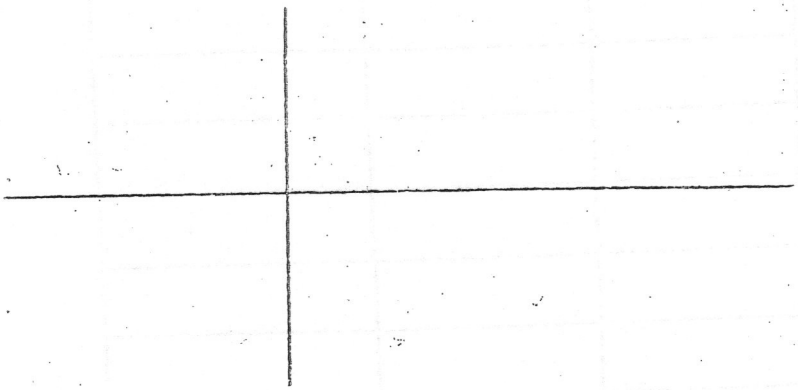
TRIAL RUN (con't)

4) $y = \cot x$



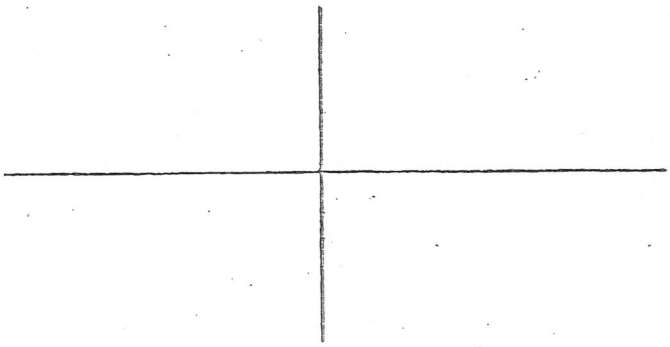
For $y = \cot x$
a) Domain _____
b) Range _____
c) Period _____
d) Zeros _____
e) Equation of assymptotes _____

5) $y = \sec x$



For $y = \sec x$
a) Domain _____
b) Range _____
c) Period _____
d) Zeros _____
e) Equation of assymptotes _____

6) $y = \csc x$



For $y = \csc x$
a) Domain _____
b) Range _____
c) Period _____
d) Zeros _____
e) Equation of assymptotes _____

TRIGONOMETRIC FUNCTIONS — TRIAL RUN (con't.)

7. Which of the trig functions are even? _____

8. Which of the trig functions are odd? _____

9. Which of the trig functions increase over the domain $(0, \frac{\pi}{2})$? _____

10. Which of the trig functions decrease over the domain $(0, \frac{\pi}{2})$? _____

11. Which trig function graphs have Y-axis symmetry? _____

12. Which trig function graphs have Origin symmetry? _____

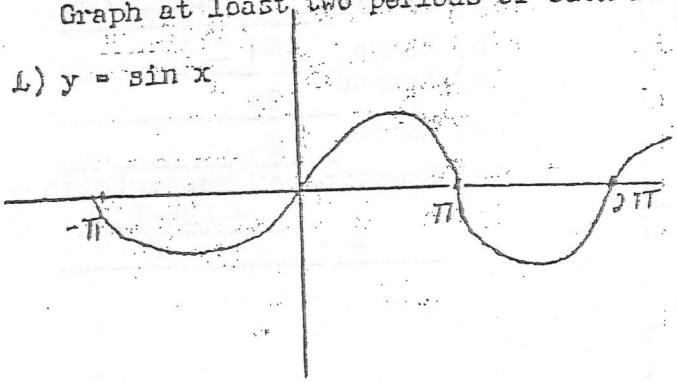
II. Complete the following table

x	sin x	cos x	tan x	sec x	csc x	cot x
$\frac{2\pi}{3}$						
0						
$\frac{5\pi}{6}$						
$-\frac{2\pi}{3}$						
$-\frac{7\pi}{6}$						
$\frac{\pi}{11}$						
$\frac{3\pi}{11}$						
$\frac{\pi}{2}$						
$\frac{37\pi}{4}$						

TRIGONOMETRIC FUNCTIONS TRIAL RUN ANSWERS

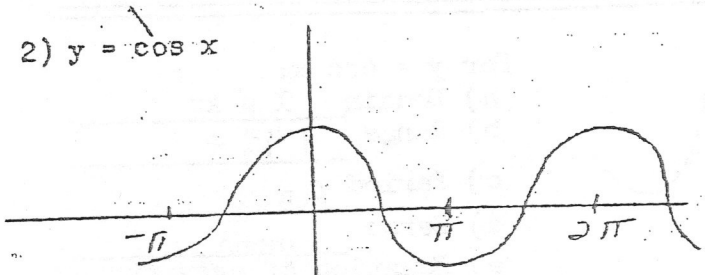
I. Graph each of the following trig functions: Label the graphs carefully.
Graph at least two periods of each function.

1) $y = \sin x$



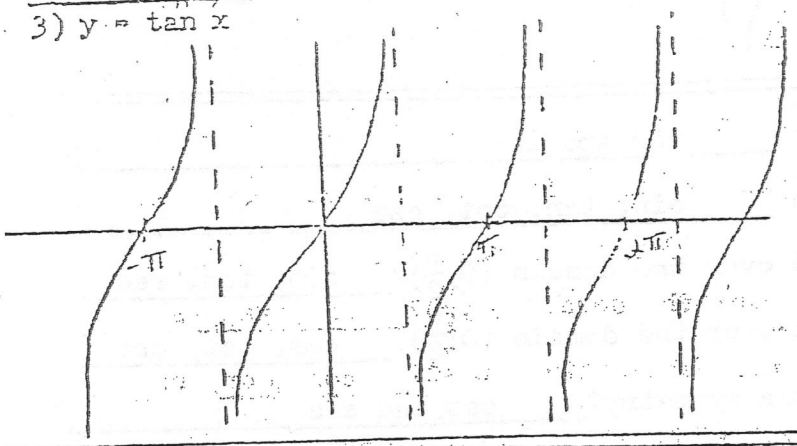
For $y = \sin x$:
a) Domain REALS
b) Range $[-1,1]$
c) Period 2π
d) Zeros $\theta = k\pi$
e) Amplitude 1

2) $y = \cos x$



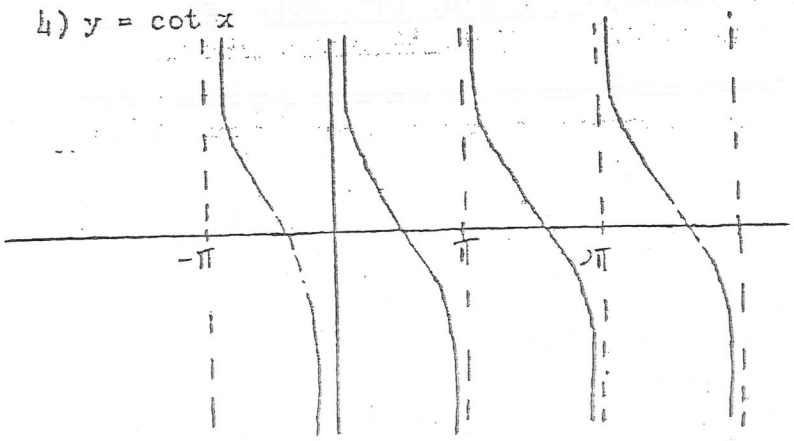
For $y = \cos x$
a) Domain REALS
b) Range $[-1,1]$
c) Period 2π
d) Zeros $\theta = \frac{(2k+1)\pi}{2}$
e) Amplitude 1

3) $y = \tan x$



For $y = \tan x$
a) Domain $x \neq \frac{(2k+1)\pi}{2}$
b) Range REALS
c) Period π
d) Zeros $\theta = k\pi$
e) Equation of assymptotes $\theta = \frac{(2k+1)\pi}{2}$

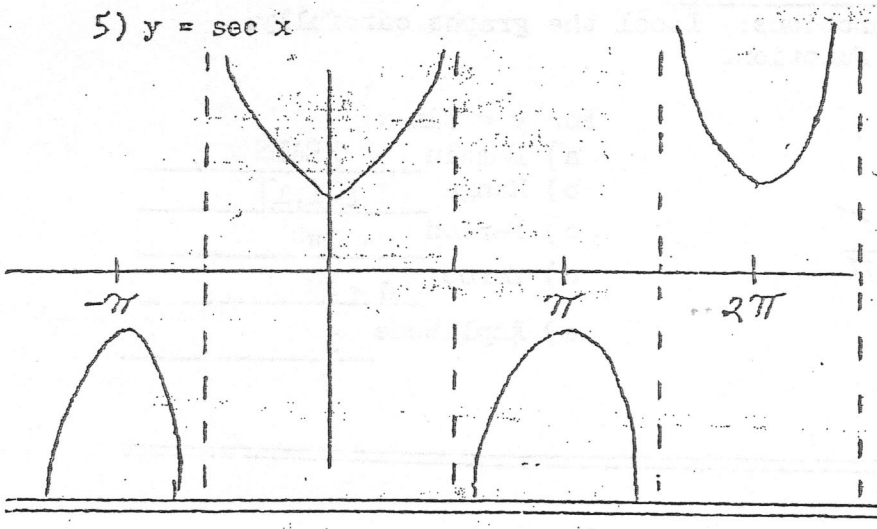
4) $y = \cot x$



For $y = \cot x$
a) Domain $x \neq k\pi$
b) Range REALS
c) Period π
d) Zeros $\theta = \frac{(2k+1)\pi}{2}$
e) Equation of assymptotes $\theta = k\pi$

TRIGONOMETRIC FUNCTIONS TRIAL RUN ANSWERS

5) $y = \sec x$



For $y = \sec x$:

a) Domain $x \neq \frac{(2k+1)\pi}{2}$

b) Range $|y| \geq 1$

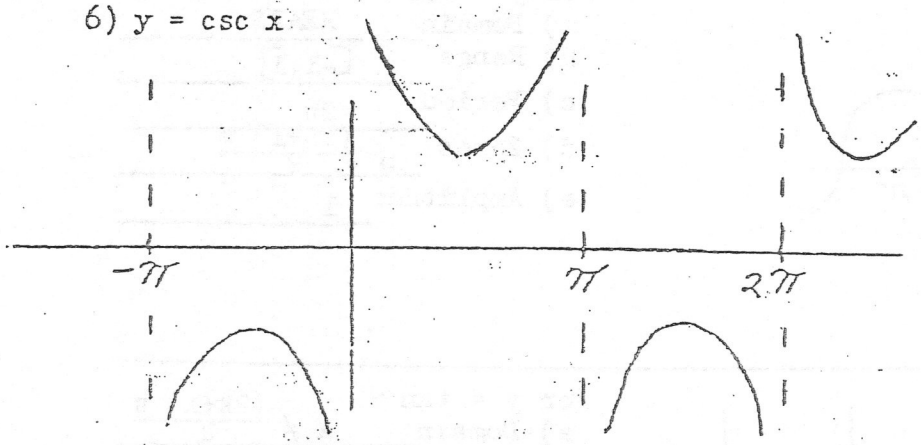
c) Period 2π

d) Zeros $\emptyset$

e) Equation of assymptotes

$\theta = \frac{(2k+1)\pi}{2}$

6) $y = \csc x$



For $y = \csc x$:

a) Domain $x \neq k\pi$

b) Range $|y| \geq 1$

c) Period 2π

d) Zeros none

e) Equation of assymptotes

$\theta = k\pi$

7. Which of the trig functions are even? cos and sec

8. Which of the trig functions are odd? sin, tan, cot, csc

9. Which of the trig functions increase over the domain $(0, \frac{\pi}{2})$? sin, tan, sec

10. Which of the trig functions decrease over the domain $(0, \frac{\pi}{2})$? cos, cot, csc

11. Which trig function graphs have Y-axis symmetry? cos and sec

12. Which trig function graphs have Origin symmetry? sin, tan, cot, csc

TRIGONOMETRIC FUNCTIONS TRIAL RUN ANSWERS

II. Complete the following table

x	sin x	cos x	tan x	sec x	csc x	cot x
$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	-2	$\frac{2\sqrt{3}}{3}$	$-\frac{\sqrt{3}}{3}$
0	0	1	0	1	---	---
$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\frac{2\sqrt{3}}{3}$	2	$-\sqrt{3}$
$-\frac{2\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$	-2	$-\frac{2\sqrt{3}}{3}$	$\frac{\sqrt{3}}{3}$
$-\frac{7\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\frac{2\sqrt{3}}{3}$	2	$-\sqrt{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	$-\sqrt{2}$	$\sqrt{2}$	-1
$-\frac{\pi}{2}$	-1	0	---	---	-1	0
$\frac{37\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	$-\sqrt{2}$	$-\sqrt{2}$	1